Representation of Fully Three-Dimensional Interfacial Curvature in the Generalised Network Model

Pore-scale Consortium Meeting 2023

12/01/2023

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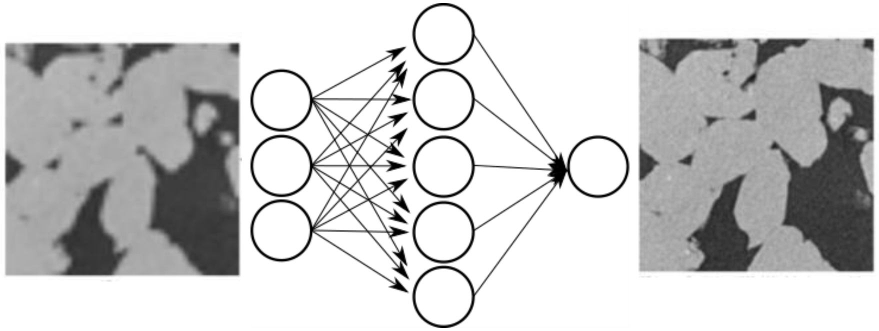
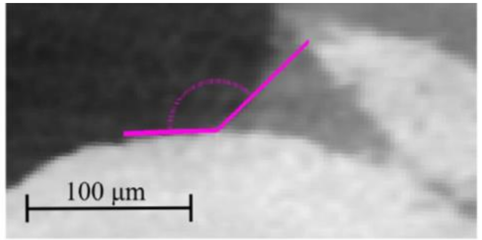
The challenge: how to predict macroscopic two-phase flow parameters



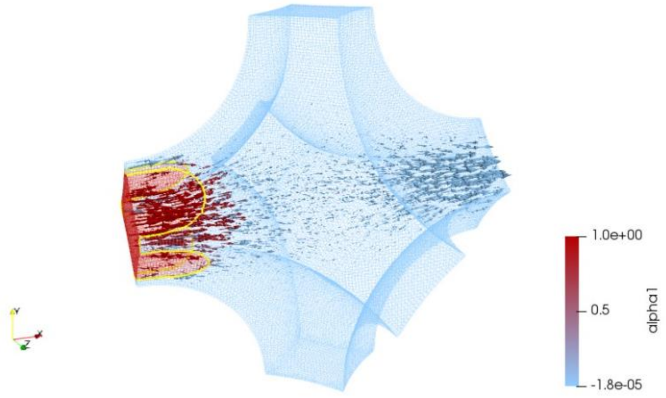
e.g, Wang et al. (2019)

High Resolution Images

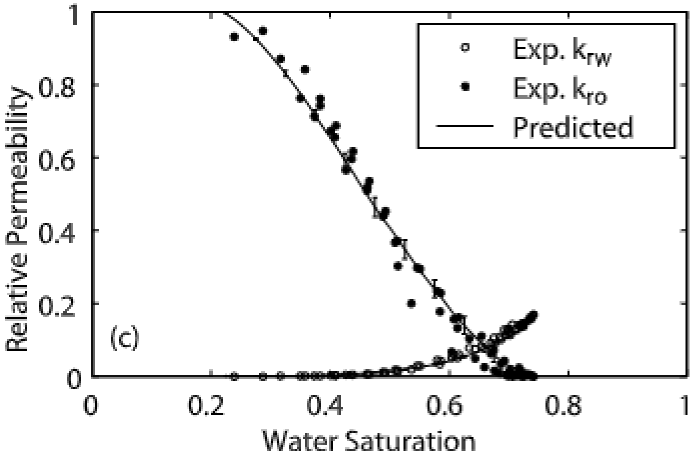


e.g, Andrew et al. (2014), Blunt et al (2019), Sun et al. (2020), Regaieg et al. (2022)

Constrained Parameters



Physics-based modelling



e.g, Valvatne and Blunt (2004)

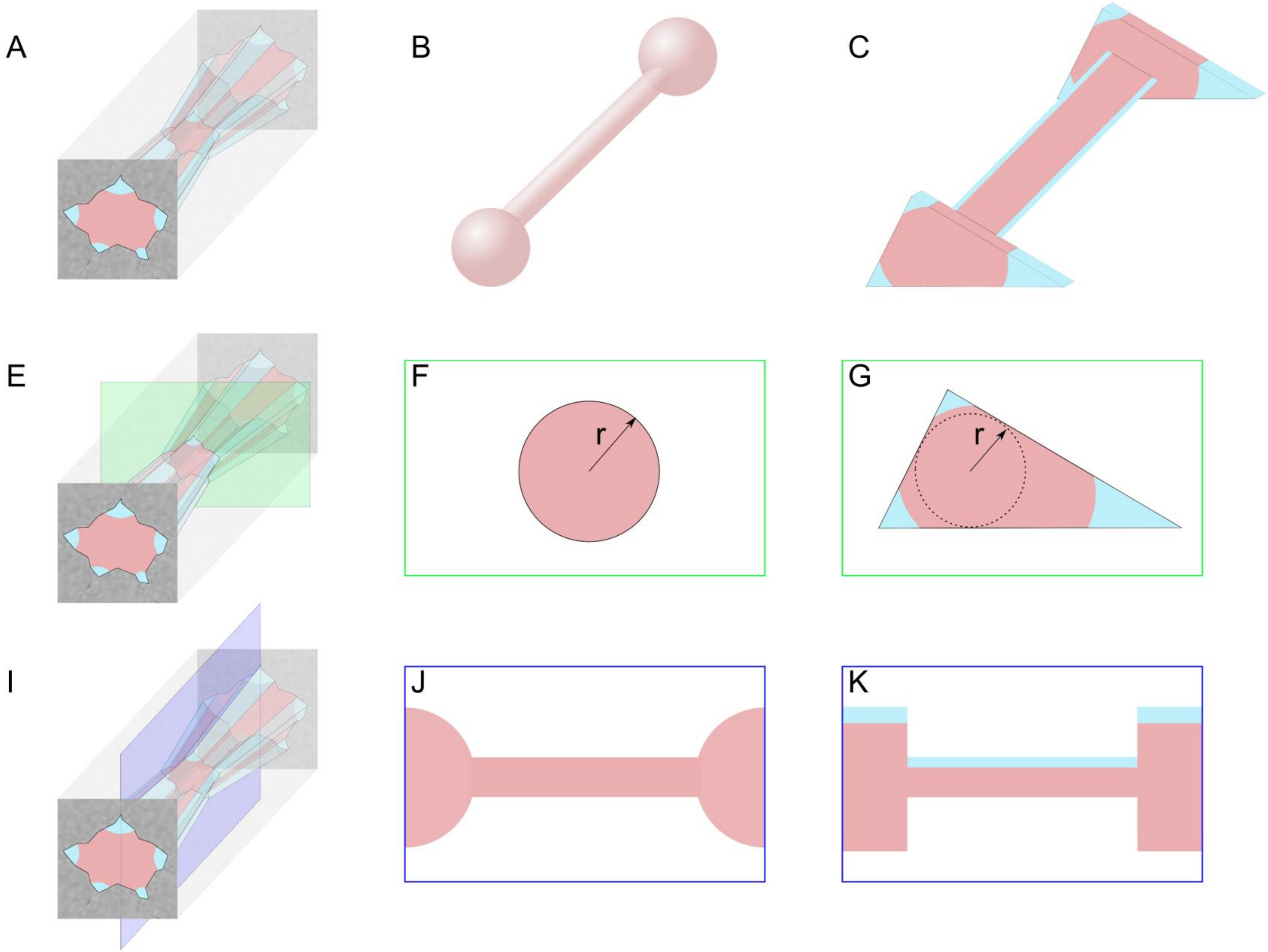
Predictive Capability

Direct Numerical Simulations?

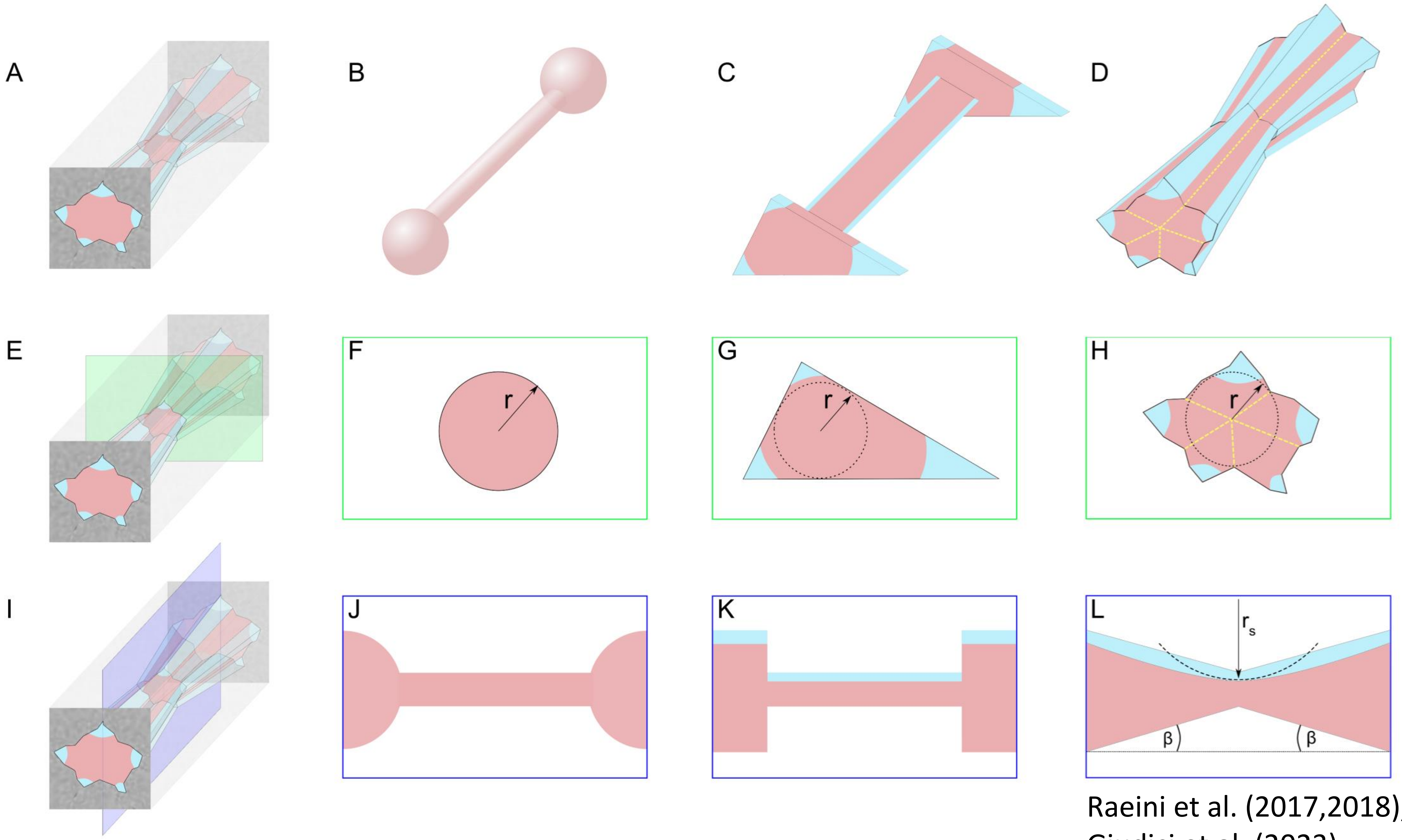
- Very demanding
- Not yet suitable for microporosity

Network models are more feasible and can be calibrated with DNS…

Current pore network models are quasi two-dimensional

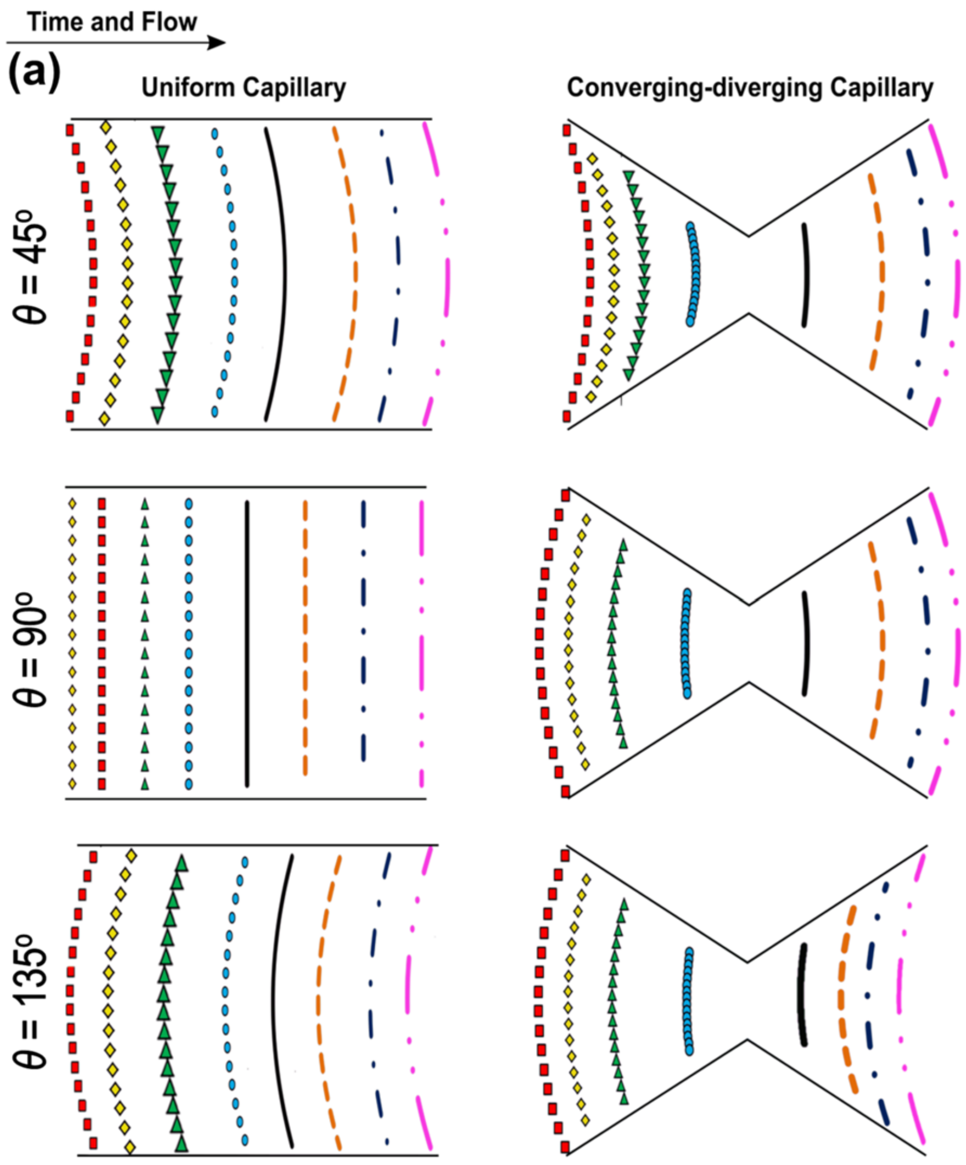


The generalised network extraction algorithm preserves 3D geometry

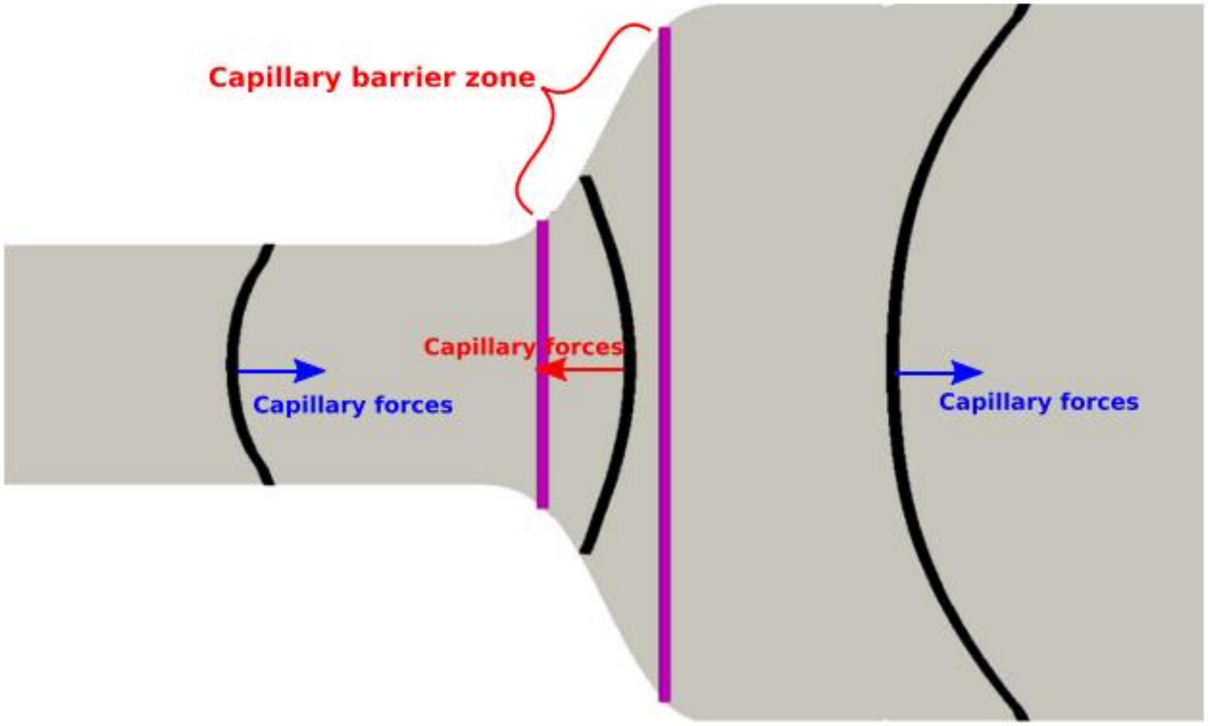


Raeini et al. (2017,2018),
Giudici et al. (2023)

What are the important 3D parameters? Pore space expansion…

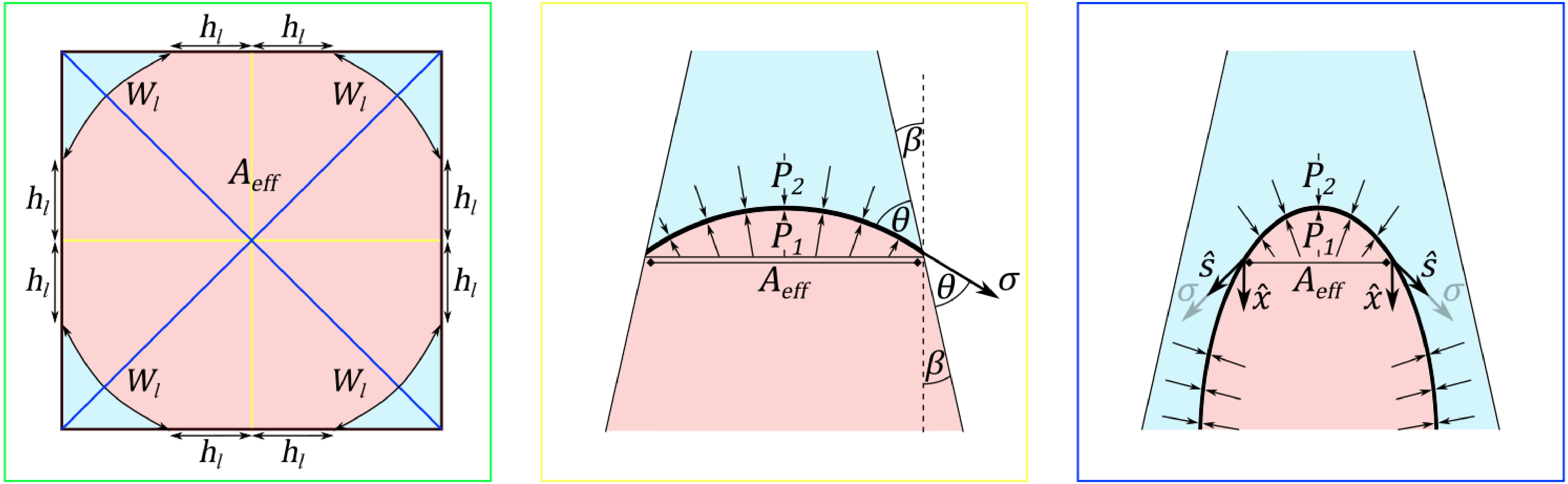


Rabbani et al. (2018)



Pavuluri et al. (2019)

What are the important 3D parameters? Pore space expansion…



$$P_1 A_{eff} = P_2 A_{eff} + \sigma 8 h_l \cos(\theta + \beta) + \sigma 4 W_l \hat{\mathbf{s}} \cdot \hat{\mathbf{x}}$$

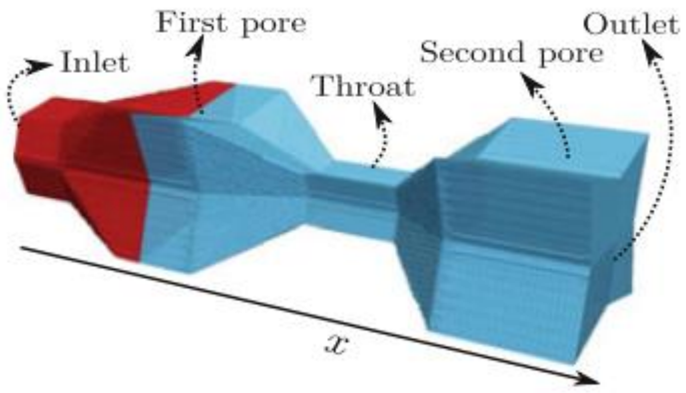
$$P_1 - P_2 = P_c = \frac{\sigma (8 h_l \cos(\theta + \beta) + 4 W_l \hat{\mathbf{s}} \cdot \hat{\mathbf{x}})}{A_{eff}}$$

$$\kappa_{pl} = \frac{\sum_{c \in t} (2 h_l \cos(\theta + \beta) + W_l \hat{\mathbf{s}} \cdot \hat{\mathbf{x}})}{A_{total} - A_{layer}}$$

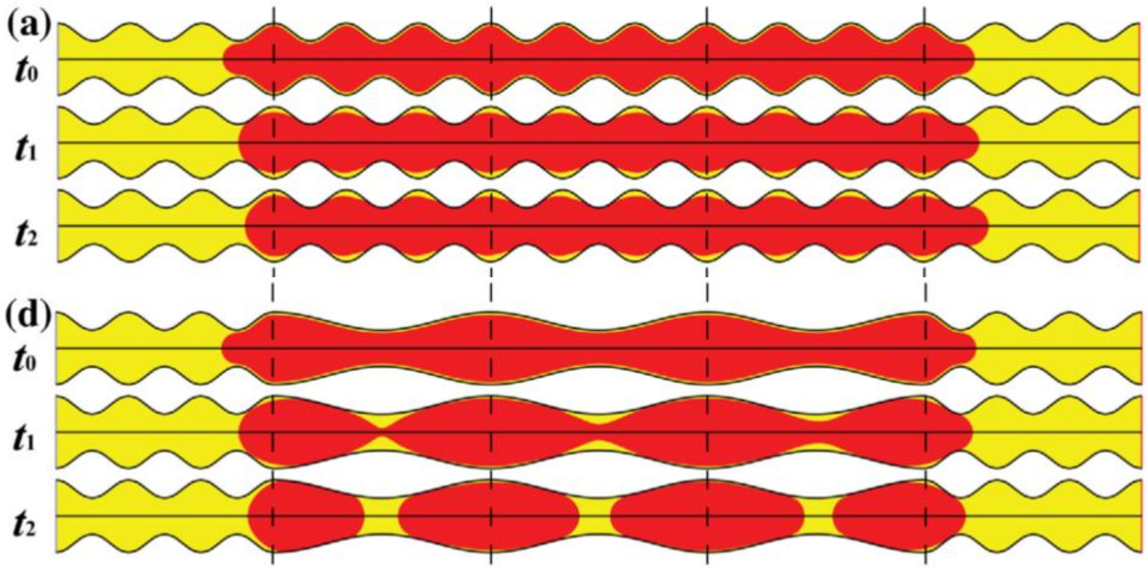
→ **This work incorporates β into the GNM**

Raeini et al (2018)

What are the important 3D parameters? Pore space expansion and sagittal curvature



Raeini et al (2014)

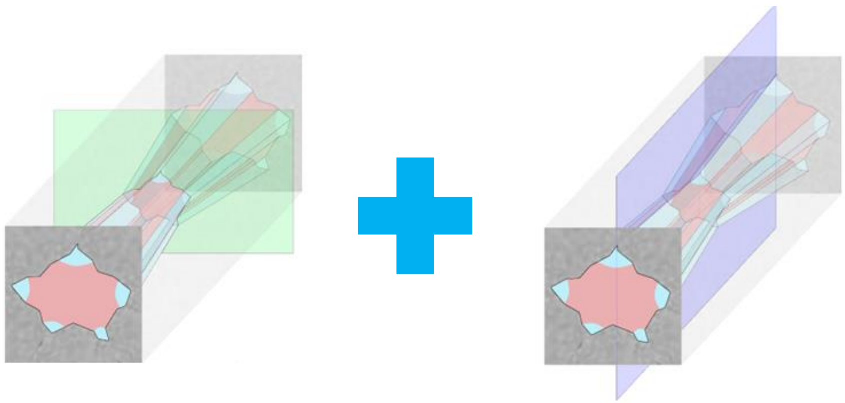


Deng et al (2014)

Key Points

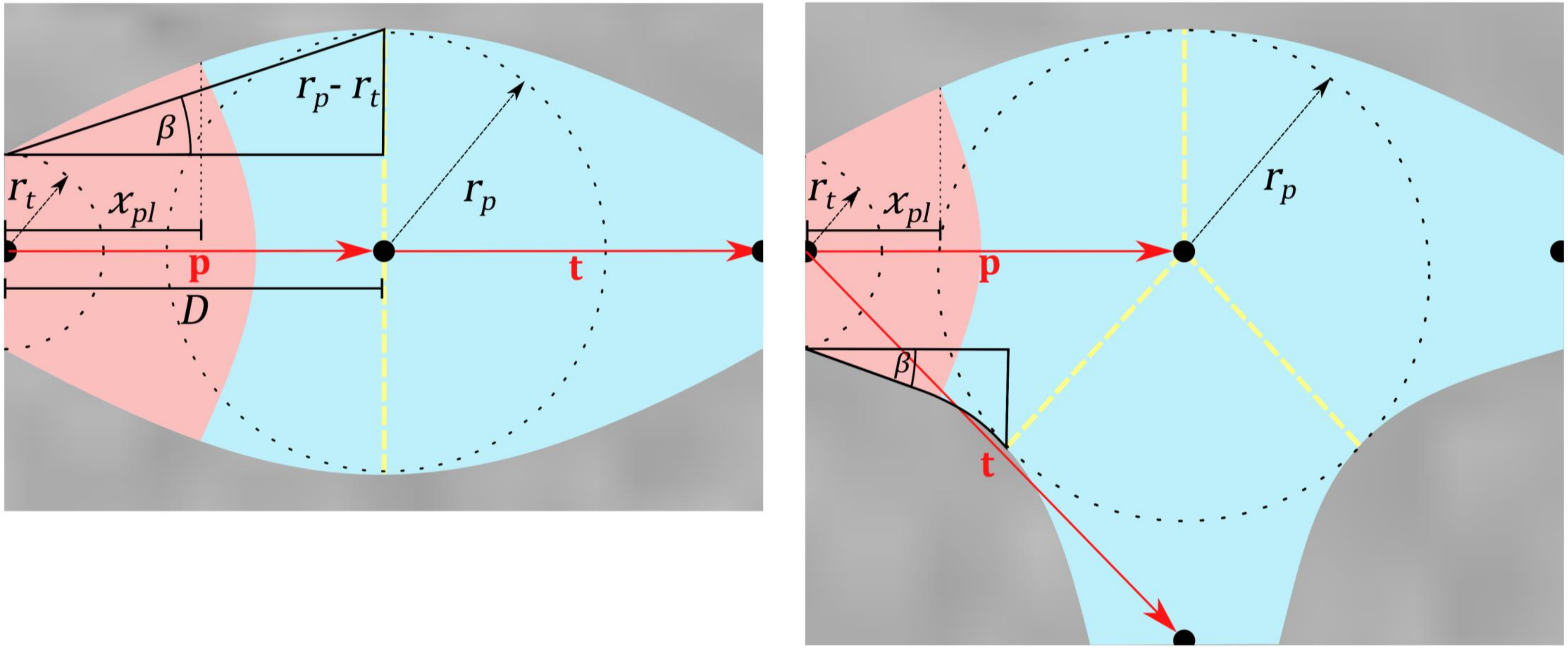
- Both studies note that the sagittal plane affects snap-off
  - Reason for this:

$$P_c = \sigma\kappa = \sigma(\kappa_a + \kappa_s)$$



- **This work quantifies and incorporates layer κ_s into the GNM**

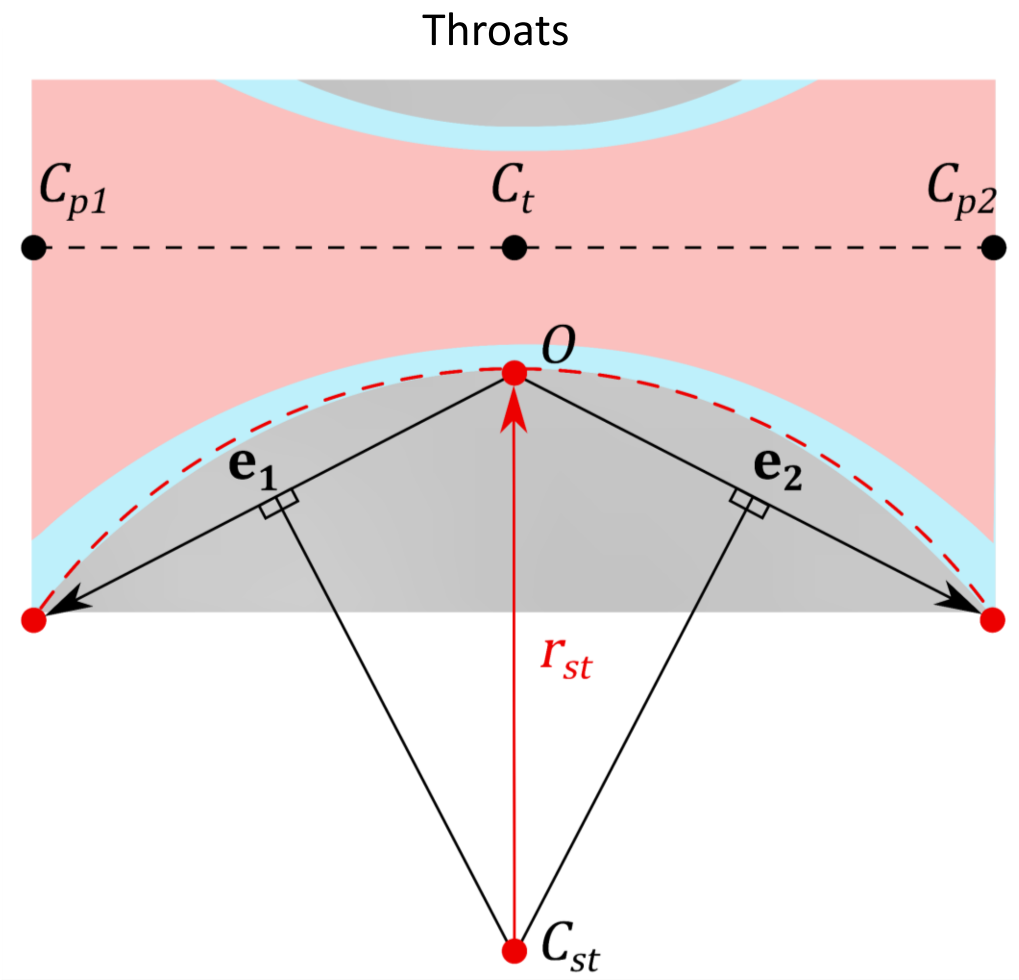
GNM developments: pore space expansion

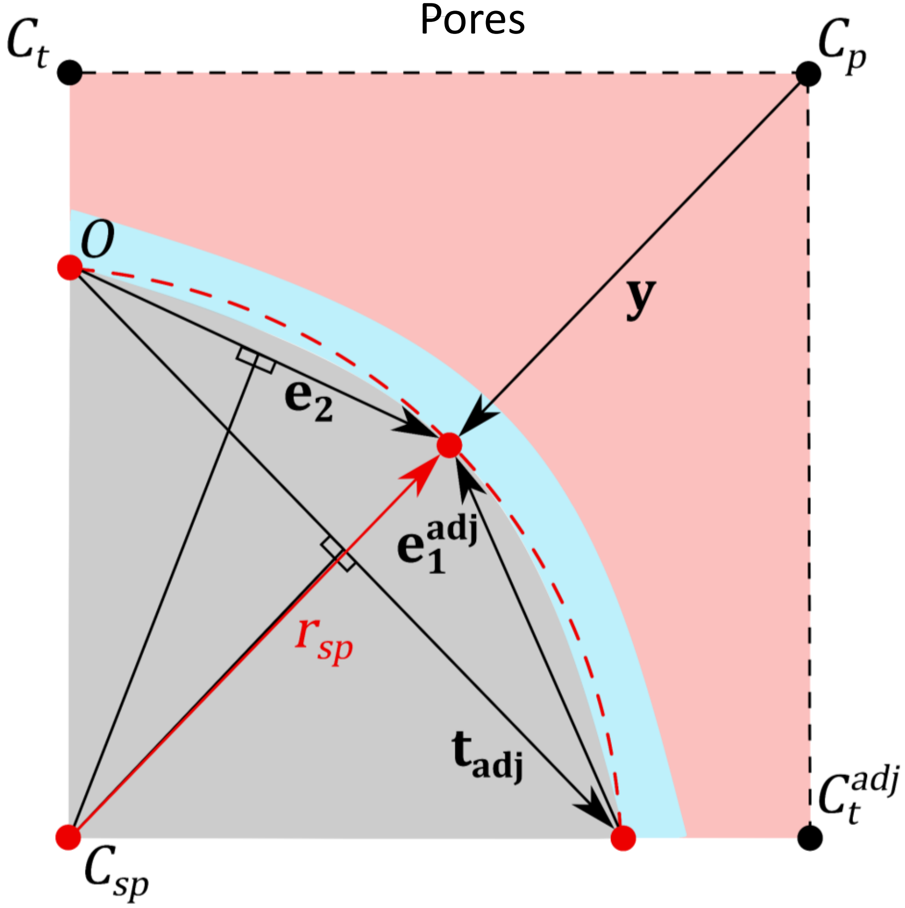


$$\beta = \tan^{-1}\left(\frac{a\sin(\pi x_{pl})(r_p - r_t)}{D}\right) + b\sin(\frac{\pi}{2}x_{pl})\cos^{-1}\left(\frac{\mathbf{t}\cdot\mathbf{p}}{\|\mathbf{t}\|\,\|\mathbf{p}\|}\right)$$

$$x_{pl} = \{0, \frac{1}{2}, 1\}$$

GNM developments: sagittal curvature of layer interfaces

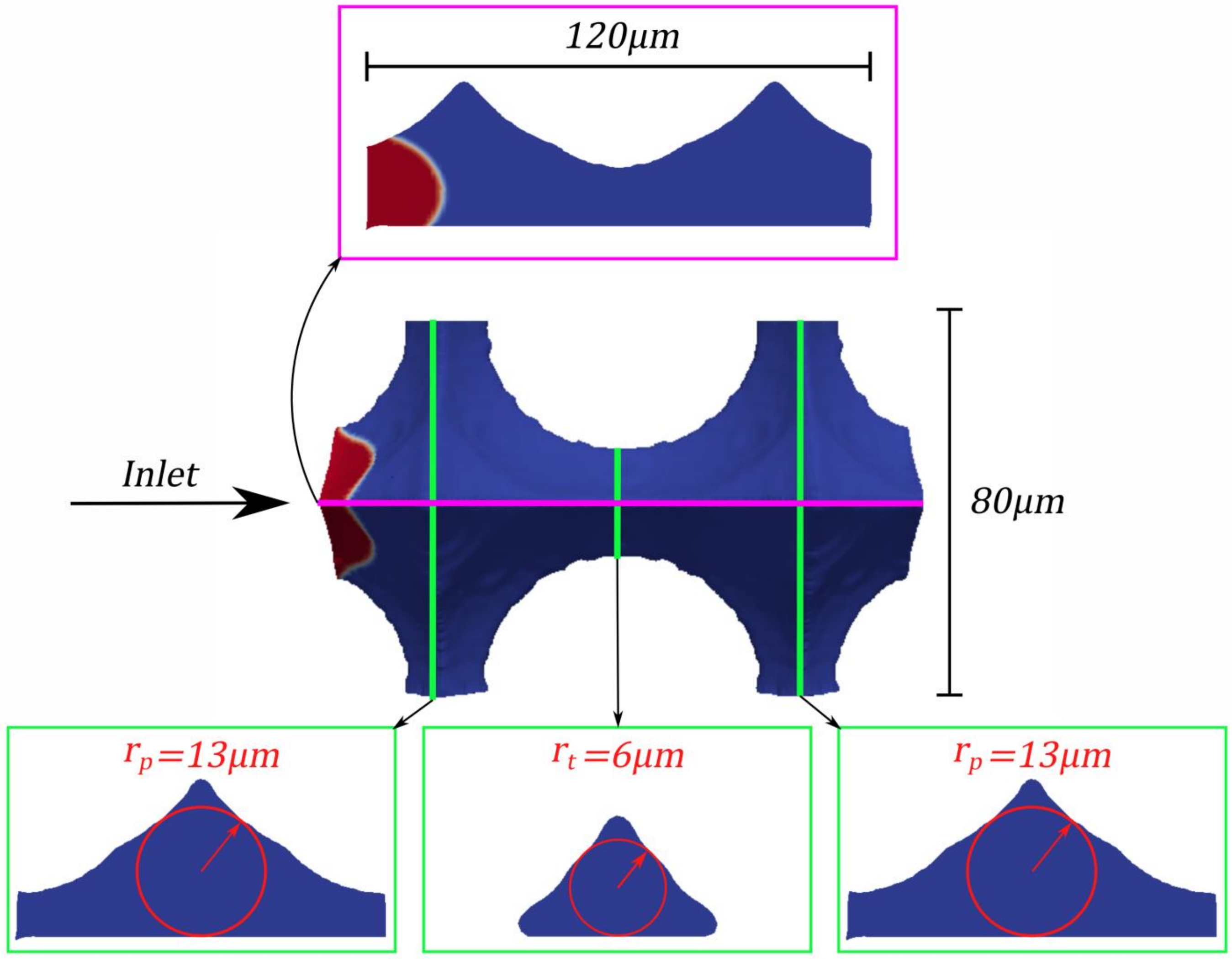




$$r_{st} = -\left|\mathbf{C_{st}}\right|$$

$$r_{sp} = \begin{cases} |\mathbf{C_{sp}}|, & (\hat{\mathbf{e}}_\mathbf{2} + \hat{\mathbf{e}}_\mathbf{1}^\mathbf{adj}) \cdot \hat{\mathbf{y}} > \mathbf{0} \\ -\left|\mathbf{C_{sp}}\right|, & \text{otherwise.} \end{cases}$$

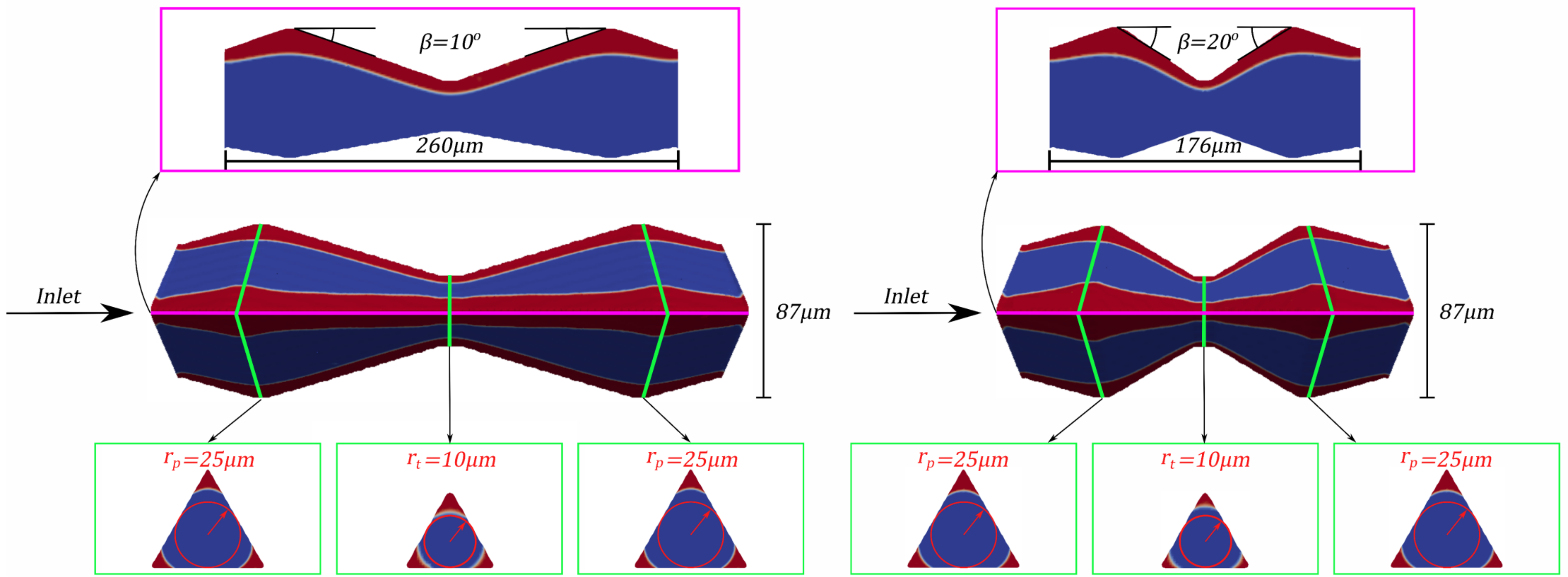
To validate and calibrate developments, piston-like advance is simulated through a synthetic geometry using volume-of-fluid…

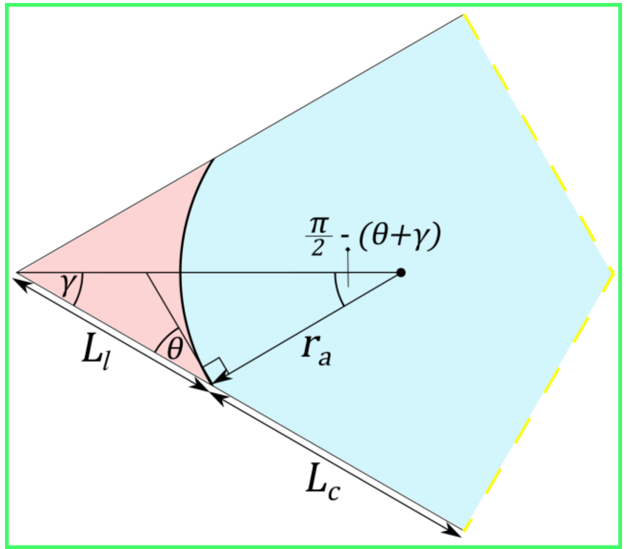


Contact Angles
30
45
60
75
90
105
120
135
150

Raeini et al. (2012), Shams et al. (2018)

and snap-off is simulated in synthetic geometries





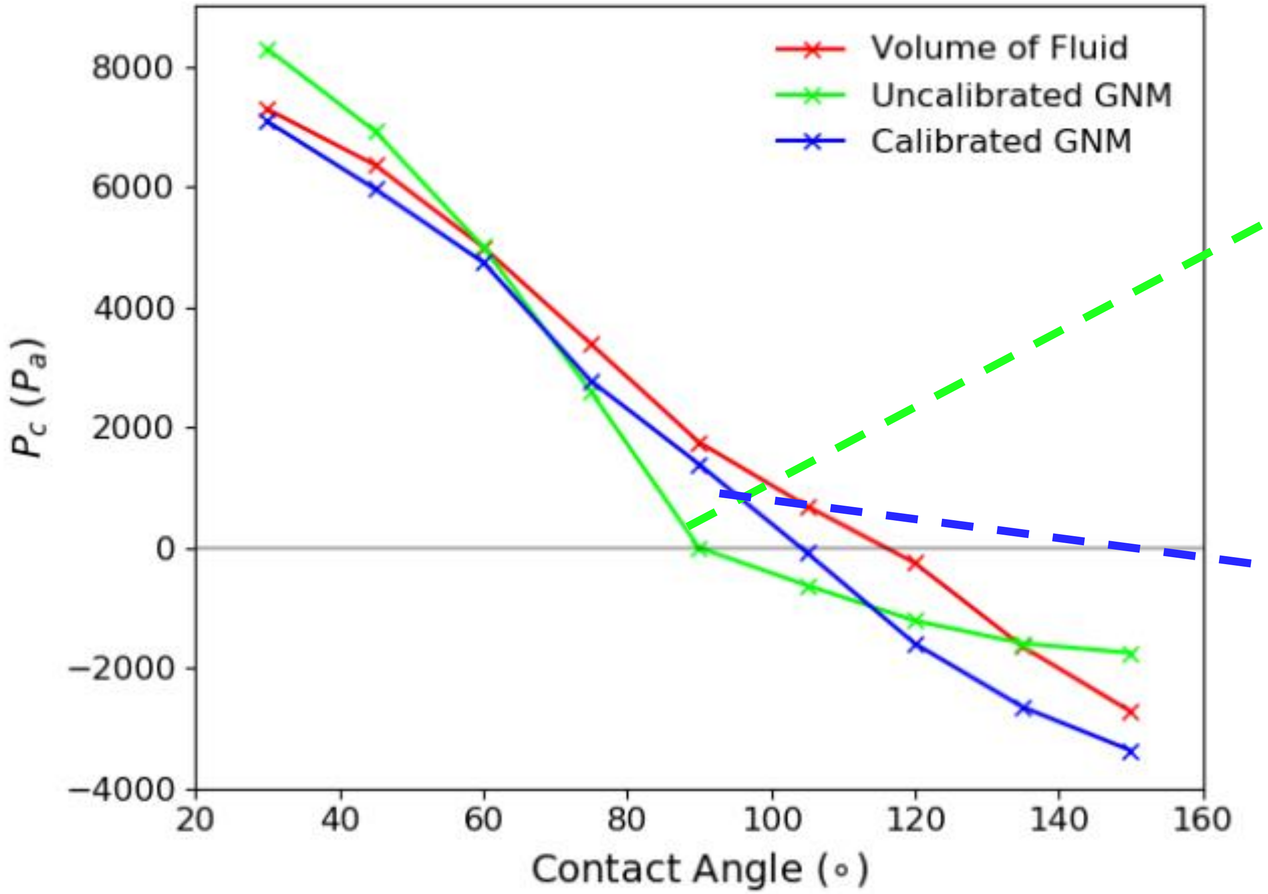
$$\frac{1}{r_a} = \frac{\cos\theta(\cot\gamma - \tan\theta)}{L_l}$$

$$= \frac{\cos\theta(\cot\gamma - \tan\theta)}{L_{tot} - L_c}$$

$$P_c = \sigma\kappa = \sigma(\kappa_a + \kappa_s)$$

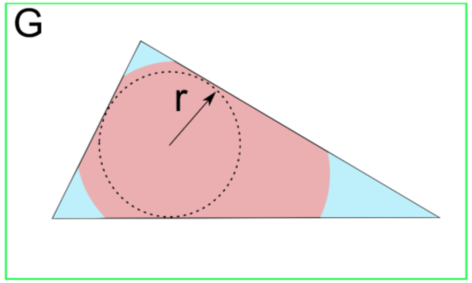
- Can determine κ_s at every time step

Accurate predictions for threshold pressure of piston-like interfaces requires the inclusion of pore-space expansion



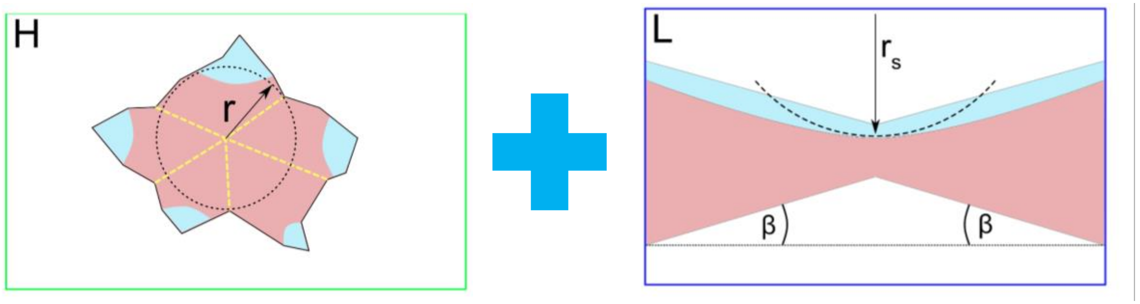
$$\kappa_{pl} = \frac{\cos\theta(1+2\sqrt{\pi G})}{r} F_d(\theta, G, \gamma)$$

Øren et al. (1998)

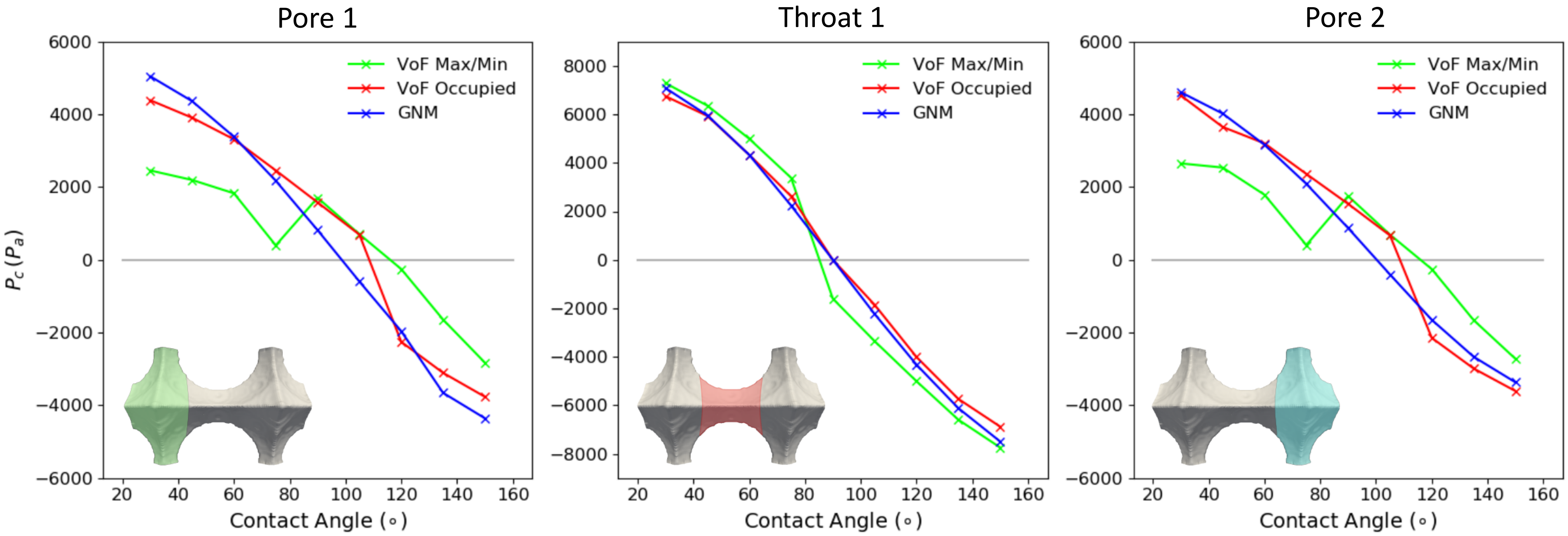


$$\kappa_{pl} = \frac{\sum\limits_{c \in t} (2h_l \cos(\theta+\beta) + W_l \hat{\mathbf{s}} \cdot \hat{\mathbf{x}})}{A_{total} - A_{layer}}$$

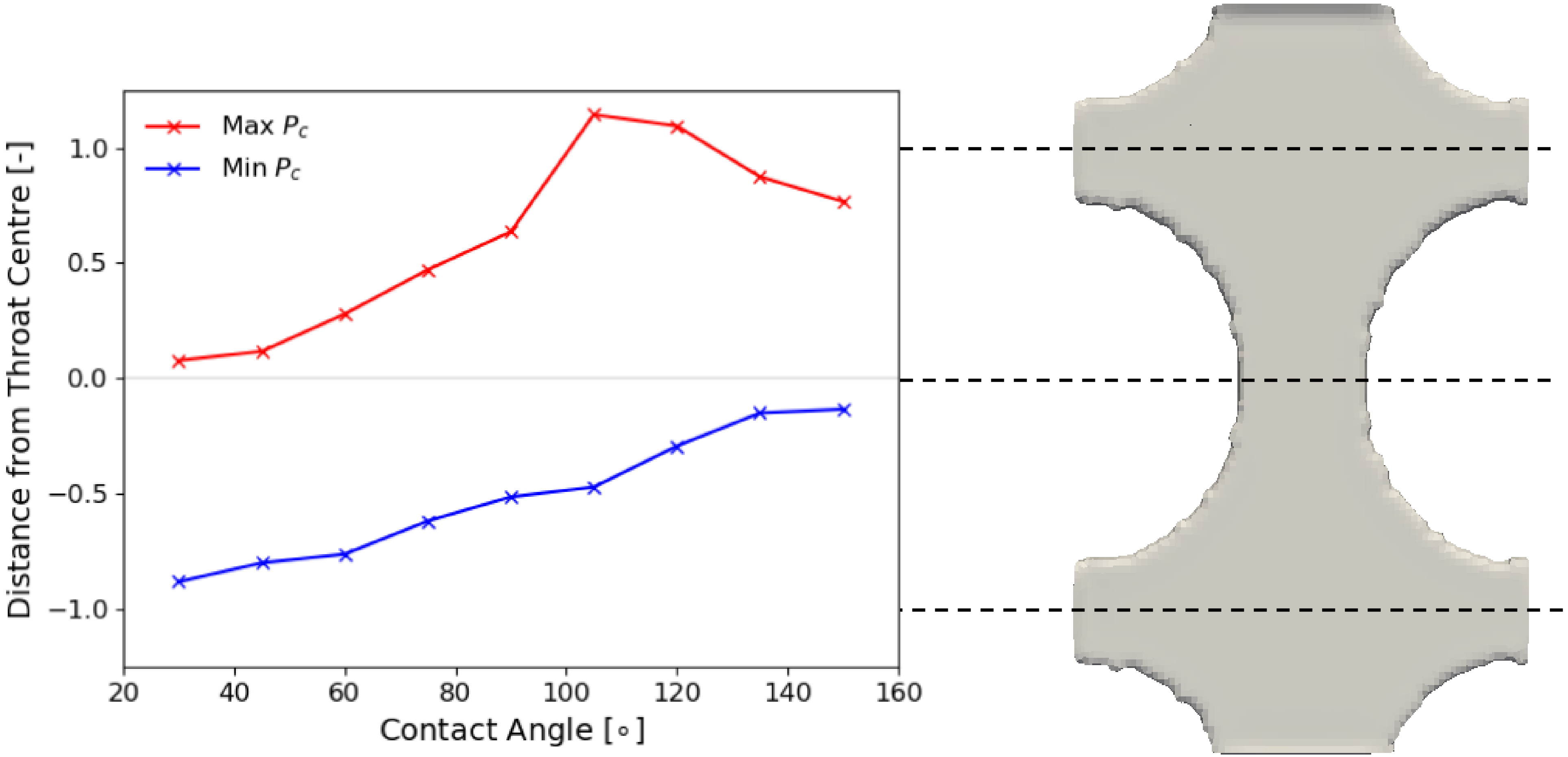
Raeini et al. (2018)



Maximum/minimum DNS pressures show disagreement with the GNM…

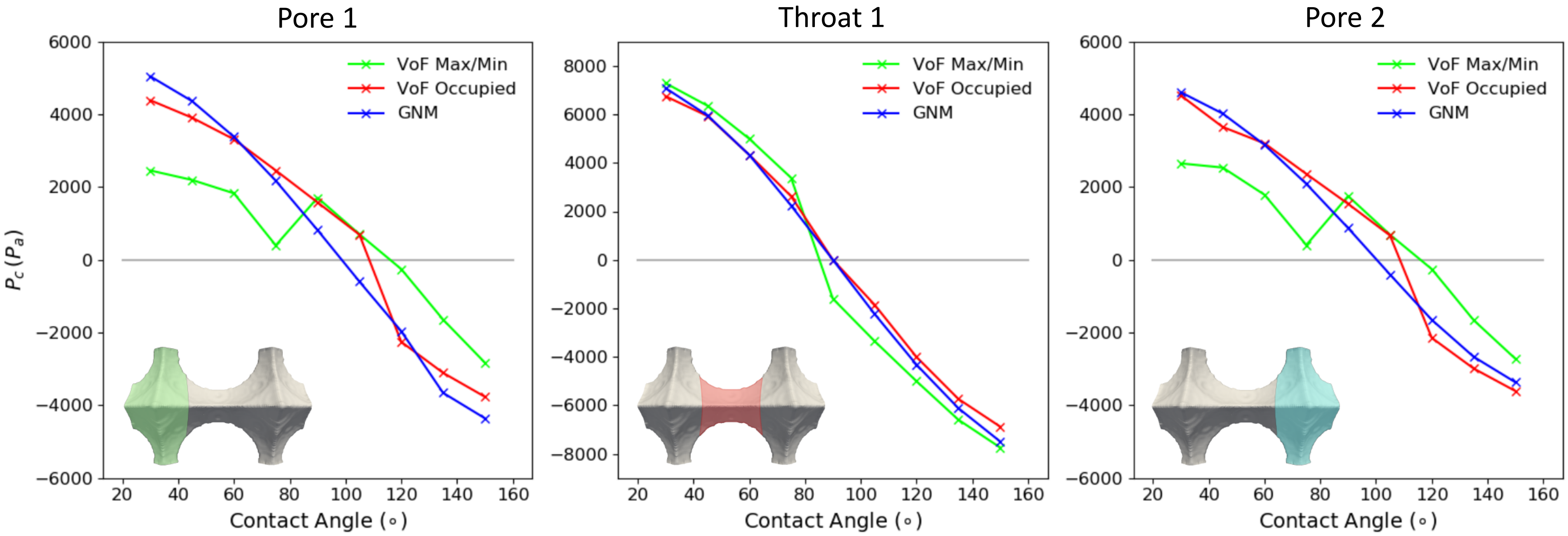


Because the position of the threshold is not always at a pore or throat centre

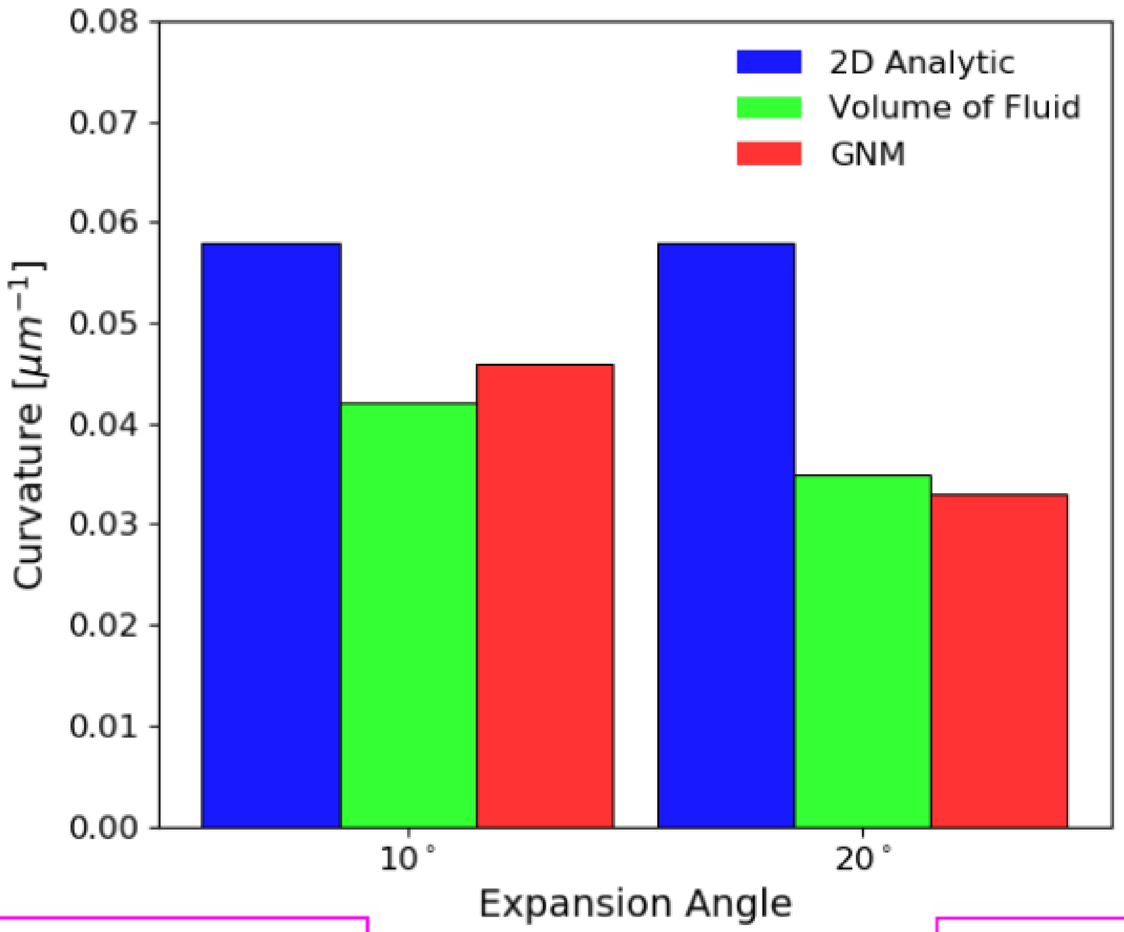


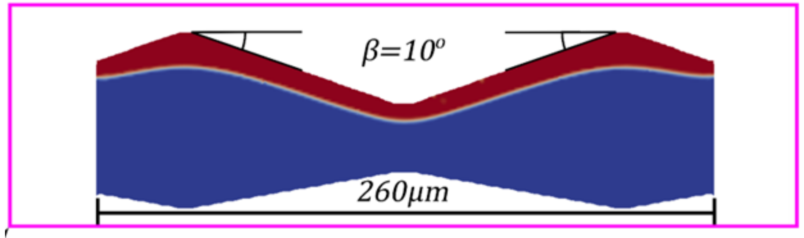
$$P_c \propto \cos(\theta + \beta)$$

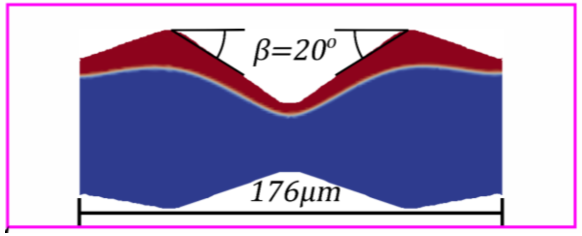
At pore/throat centres, GNM thresholds agree well with the direct method



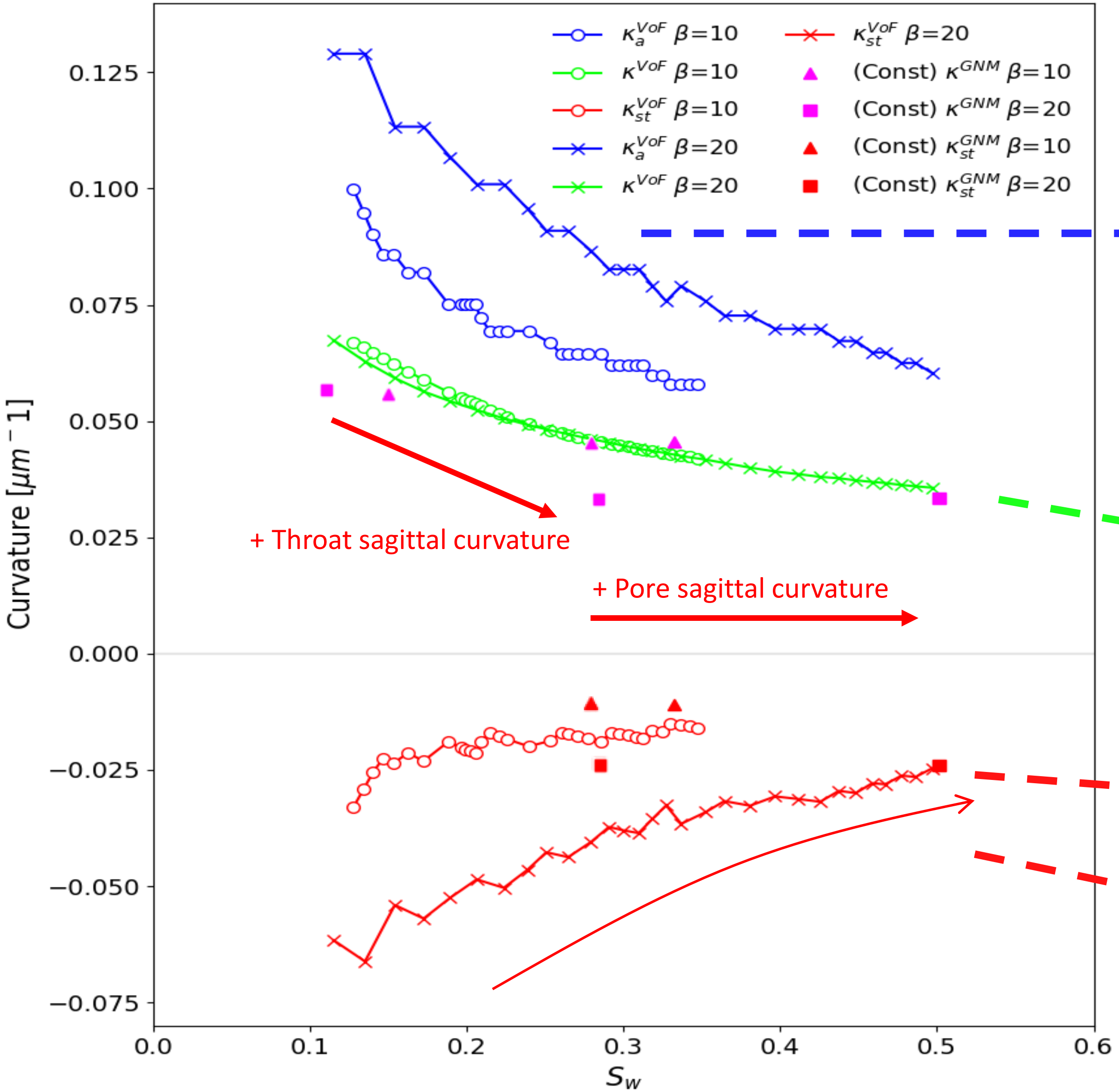
2D analytic solutions (no sagittal curvature) cannot reproduce DNS predictions for threshold snap-off curvature at different expansion angles - the GNM can

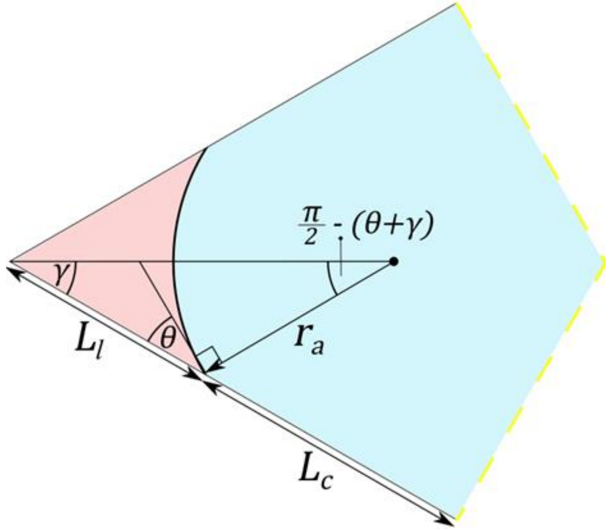






To be fully predictive, saturation must be considered. We find sagittal curvature varies with pressure.





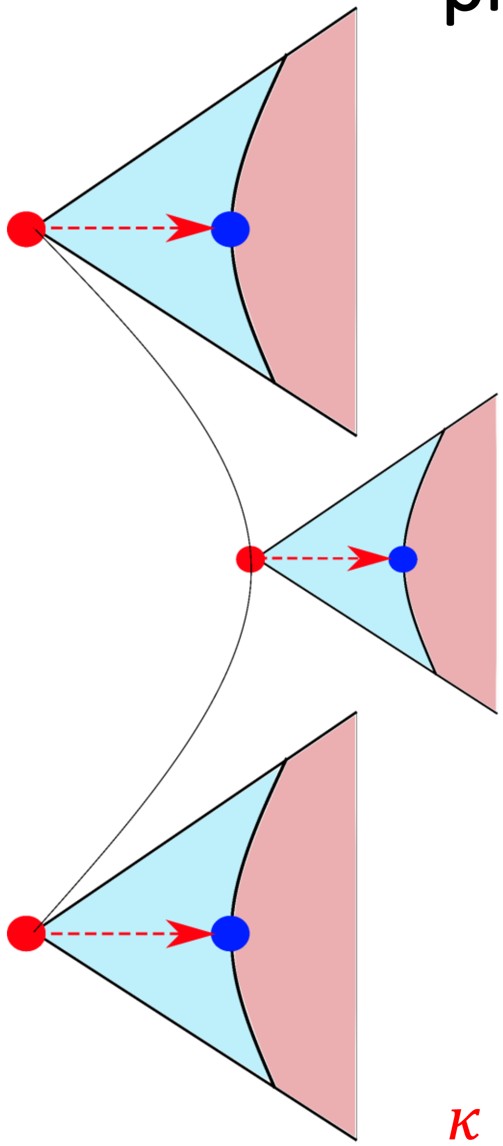
$$\kappa_a$$

$$\kappa = \frac{P_c}{\sigma} = (\kappa_a + \kappa_s)$$

$$\kappa_s = (\kappa - \kappa_a)$$

Requires an iterative method…

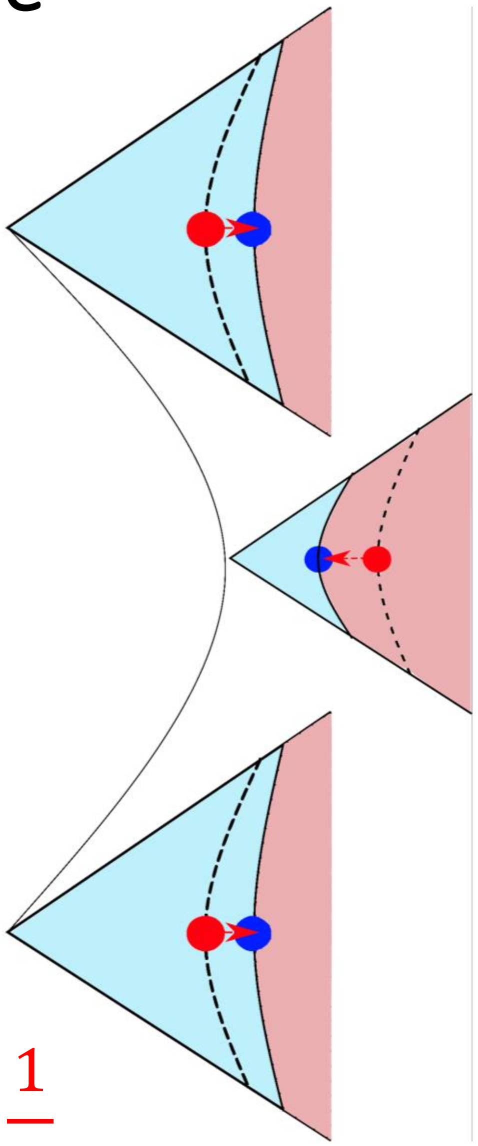
An iterative method to determine sagittal curvature as a function of pressure



Iteration 1

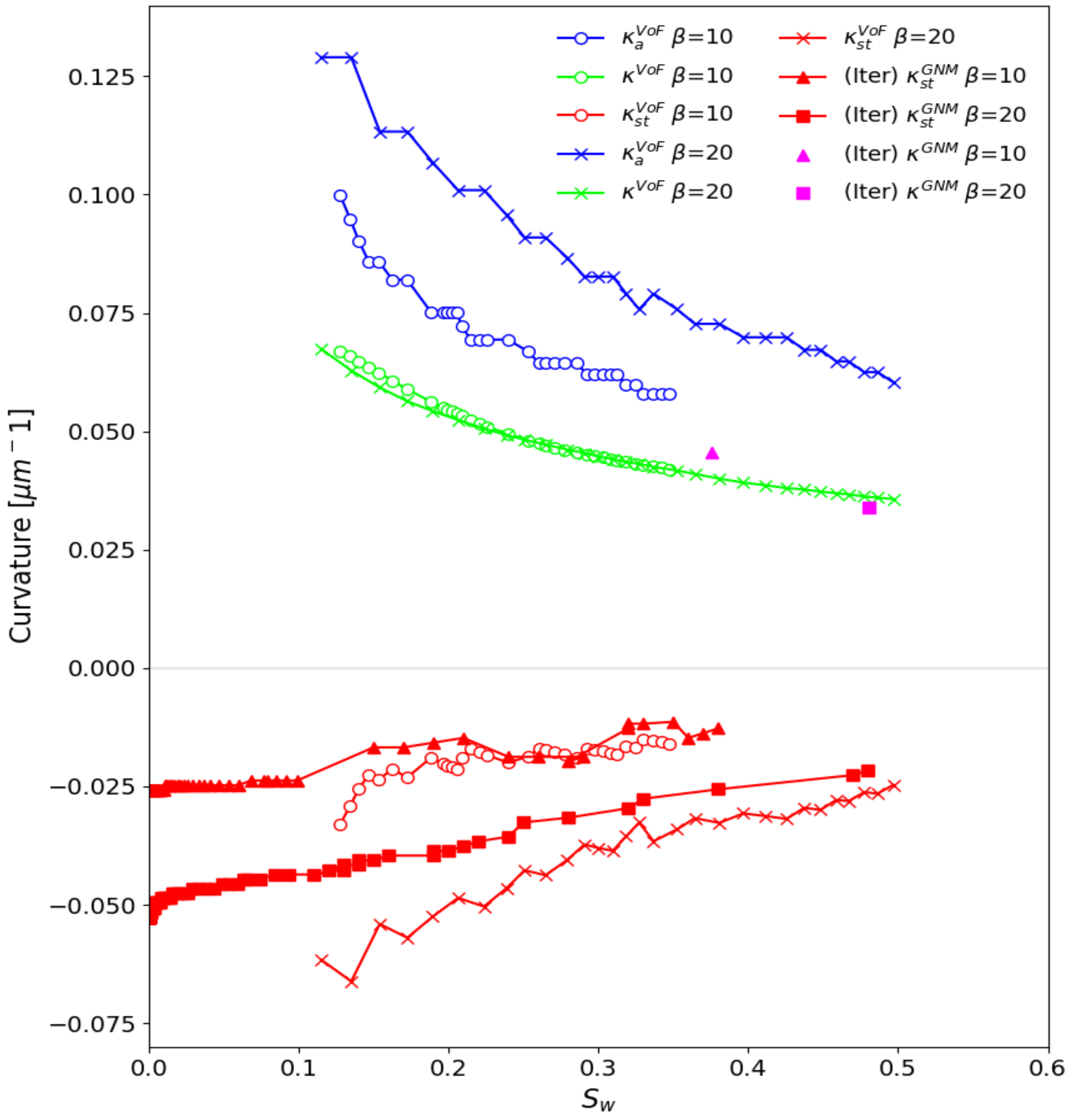
$$\kappa = \frac{1}{r_a} + \frac{1}{r_s}$$

is the same!

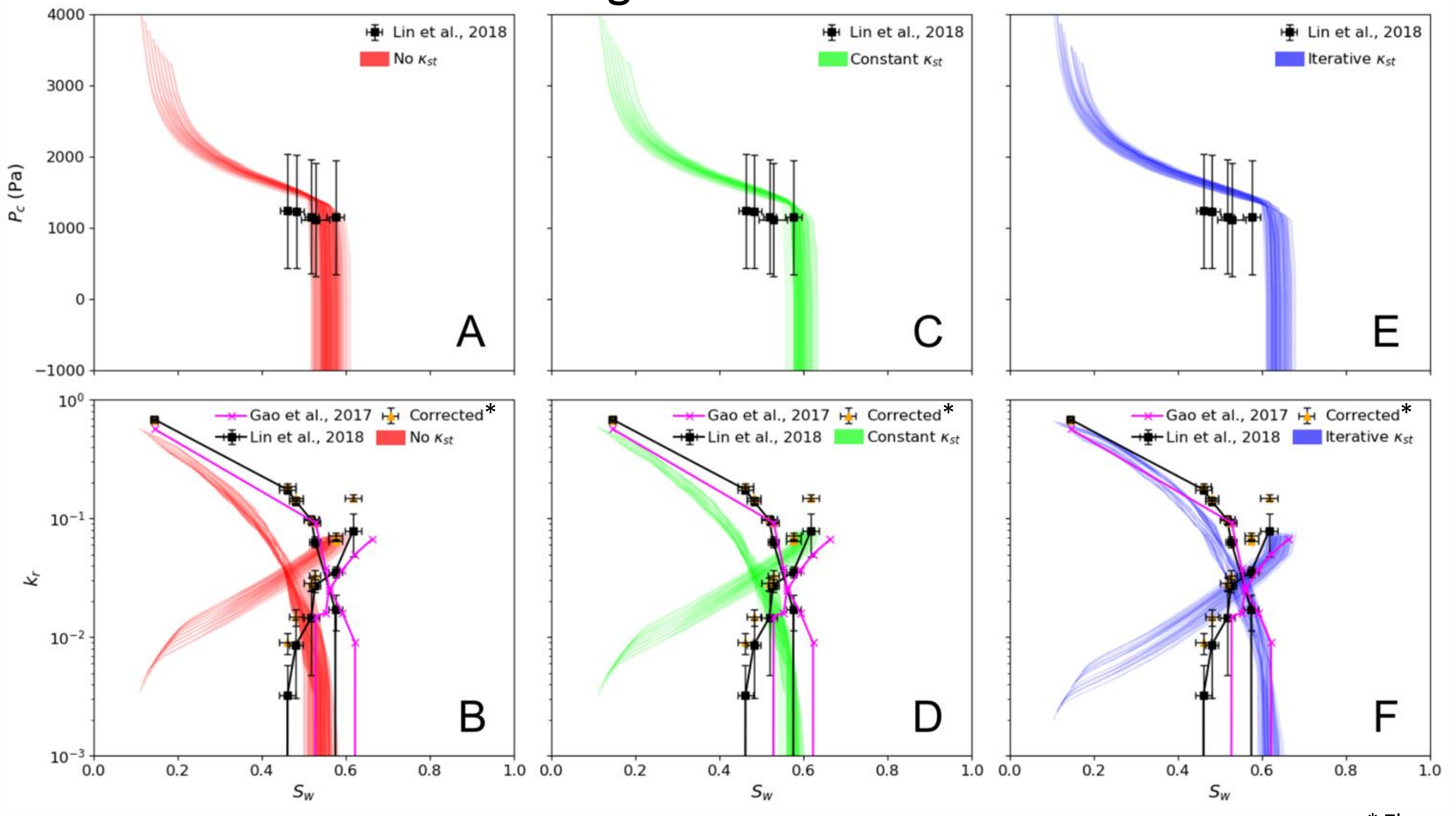


Iteration 2

The iterative method replicates DNS curvature predictions well

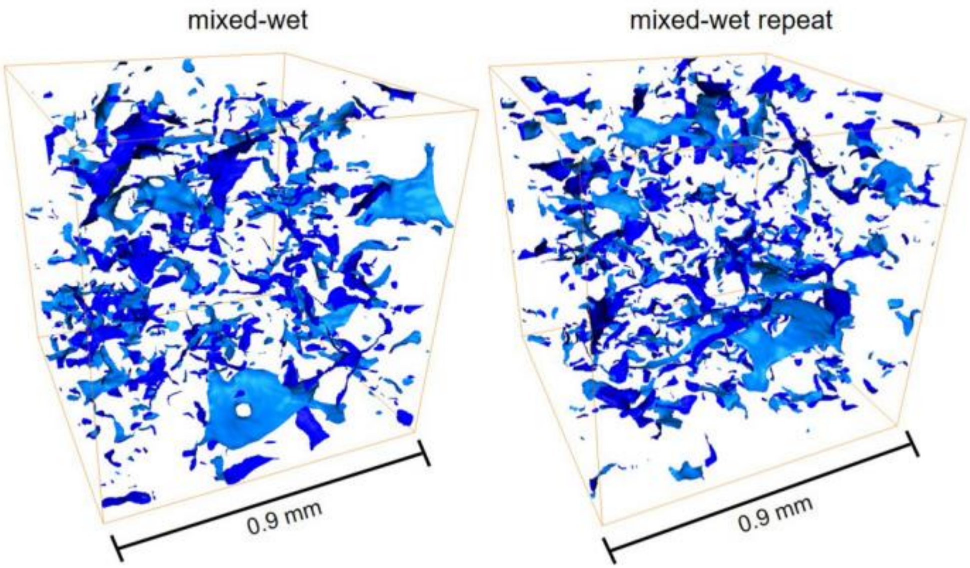


Effect of Sagittal Curvature on WW Bentheimer – A translation to higher saturations

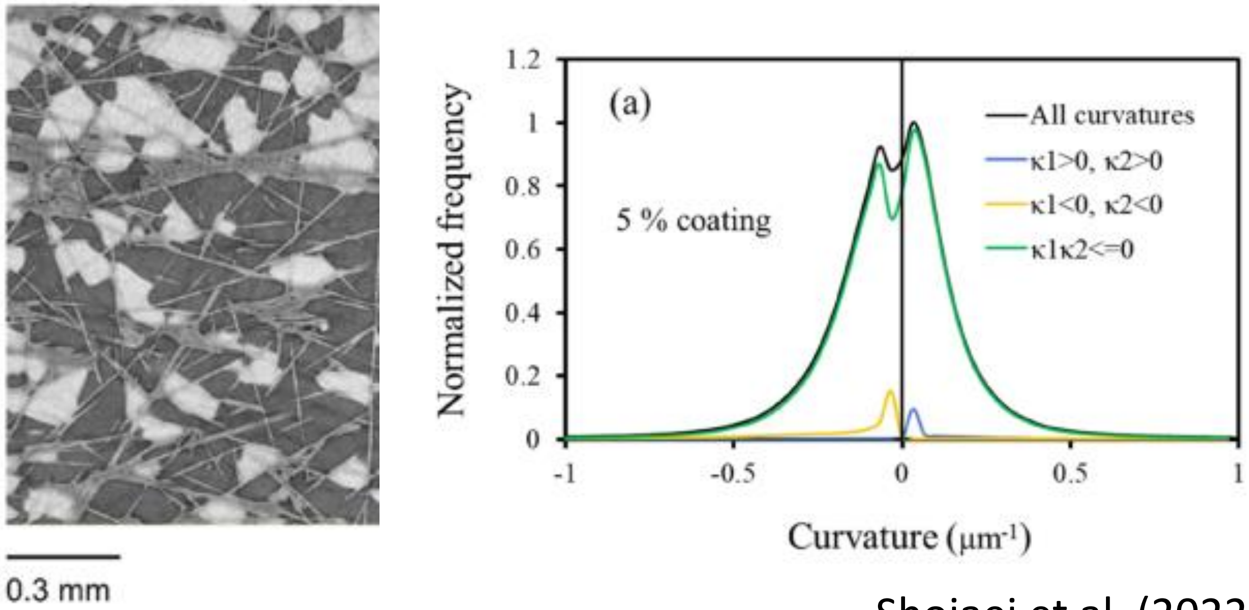


* Zhang et al (2023)

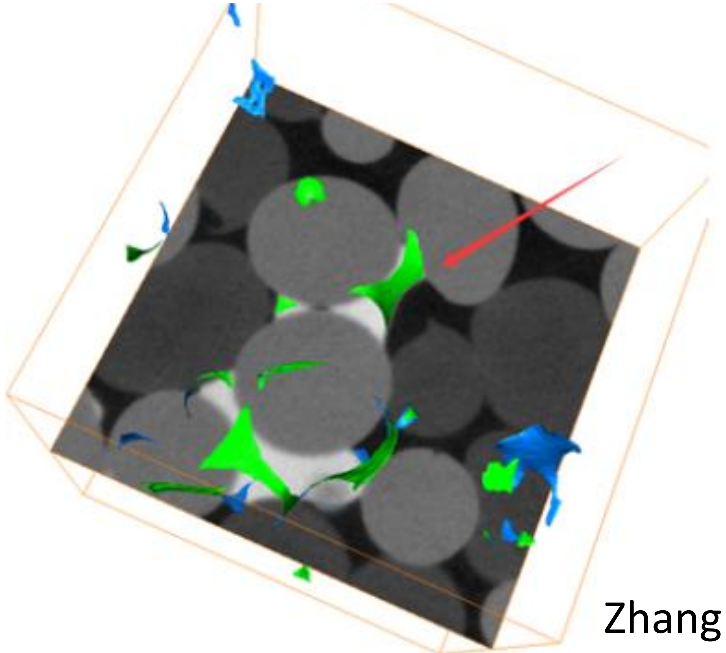
Future Directions – Mixed Wettability



Lin et al. (2019)



Shojaei et al. (2022)

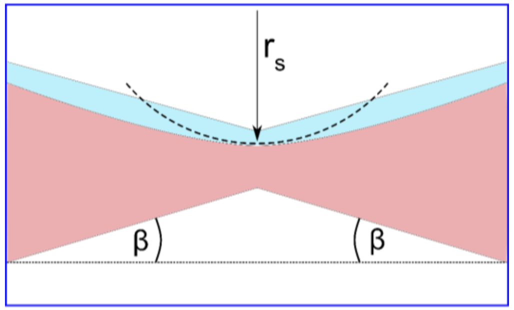


Zhang et al. (2023, in prep)

- Minimal surfaces are everywhere in mixed-wet media
- **To date, no network model has piston-like interfaces with differing principal radii of curvature**

Engineering and Physical Sciences Research Council

Conclusions



- Traditional network models are quasi-2D. A complete, 3D representation of interface curvature is needed for accurate predictions.
- This is now present in the GNM.
- Incorporation of pore-space expansion significantly improves Pc predictions.
- Sagittal curvature plays an important role in snap-off pressures, residual saturations and relative permeabilities.
- Future work aims to extend enhanced representation to mixed-wet media.